Homework 4: Due Friday, April 16

Exercises

Section 2.5: #1, 7, 8, 10, 11, 15, 16.

Section 2.6: #4, 5.

Problems

Problem 1: (From Problem 14 in Section 2.5). Let $f_1, f_2, \dots$ be a sequence of polynomials in $k[x_1, \dots, x_n]$. Show that there exists $m \in \mathbb{N}^+$ such that $f_\ell \in \langle f_1, \dots, f_m \rangle$ for all $\ell \in \mathbb{N}^+$.

Problem 2: (From Problem 13 in Section 2.5). Let $V_1 \supseteq V_2 \supseteq V_3 \supseteq \dots$ be a descending sequence of varieties in k^n. Show that there exists $m \in \mathbb{N}^+$ such that $V_\ell = V_m$ for all $\ell \geq m$.

Problem 3: (From Problem 8 in Section 2.6). Let $g_1, g_2 \in k[x_1, \dots, x_n]$ be nonzero polynomials, let $c_1, c_2 \in k$, and let $\alpha_1, \alpha_2 \in \mathbb{N}^n$. Suppose that $c_1 x^{\alpha_1} g_1$ and $c_2 x^{\alpha_2} g_2$ have the same multidegree δ. Show that $S(c_1 x^{\alpha_1} g_1, c_2 x^{\alpha_2} g_2) = x^{\delta - \gamma} \cdot S(g_1, g_2)$, where $\gamma = \operatorname{lcm}(LM(g_1), LM(g_2))$.
Note: This fact plays an important role in the proof of Theorem 2.6.6.