Homework 3: Due Tuesday, April 13

Exercises

Section 2.2: #1, 3, 5, 13.

Section 2.3: #1, 6, 9.

Section 2.4: #1, 5, 10.

Problems

Problem 1: Let $<$ be a monomial ordering on $k[x_1, \dots, x_n]$. Show that if $f, g \in k[x_1, \dots, x_n]$ are nonzero, then $LT(f \cdot g) = LT(f) \cdot LT(g)$.

Problem 2: Determine (with proof) whether the following orderings are monomial orderings:
a. Given $\alpha = (a_1, \dots, a_n)$ and $\beta = (b_1, \dots, b_n)$, define $\alpha <_{maxlex} \beta$ if either $\max\{a_i : 1 \leq i \leq n\} < \max\{b_i : 1 \leq i \leq n\}$, or $\max\{a_i : 1 \leq i \leq n\} = \max\{b_i : 1 \leq i \leq n\}$ and $\alpha <_{lex} \beta$.
b. Given $\alpha = (a_1, \dots, a_n)$ and $\beta = (b_1, \dots, b_n)$, define $\alpha <_{ilex} \beta$ if the leftmost nonzero entry of $\alpha - \beta$ is positive (i.e. if i is the first position where $a_i \neq b_i$, then $a_i > b_i$).

Problem 3: (From Problem 9 in Section 2.3).
a. Using Division with Remainder (with an appropriate monomial ordering), show that for every polynomial $f \in \mathbb{R}[x, y, z]$, there exists $g_1, g_2, r \in \mathbb{R}[x, y, z]$ with $f = g_1 \cdot (y - x^2) + g_2 \cdot (z - x^3) + r$ and where variables y and z do not occurs in r.
b. Find an explicit example of $g_1, g_2 \in \mathbb{R}[x, y, z]$ with $z^2 - x^4 y = g_1 \cdot (y - x^2) + g_2 \cdot (z - x^3)$.