Homework 1: Due Tuesday, April 6

Exercises

Section 1.1: #2, 5, 6.

Section 1.2: #1, 2, 4, 6.

Problems

Problem 1: (From Problems 3 and 4 in Section 1.1) Let F be a finite field with q elements.
a. Explain why $F\backslash\{0\}$ is a group under multiplication.
b. Show that $a^{q-1} = 1$ for all $a \in F\backslash\{0\}$.
c. Show that $a^q = a$ for all $a \in F$.
Note: This problem generalizes Fermat's Little Theorem from $\mathbb{Z}/p\mathbb{Z}$ to any finite field F.

Problem 2: Working in the ring $\mathbb{Z}[x]$, let I be the ideal

$$I = \langle 2, x\rangle = \{g(x) \cdot 2 + h(x) \cdot x : g(x), h(x) \in \mathbb{Z}[x]\}.$$

Show that I is not a principal ideal in $\mathbb{Z}[x]$.

Problem 3: Working in the ring $\mathbb{Z}[x]$, let $I = \langle x\rangle = \{g(x) \cdot x : g(x) \in \mathbb{Z}[x]\}$.
a. Show that I is the set of all elements of $\mathbb{Z}[x]$ whose constant term is 0.
b. Show that I is a prime ideal.
c. Show that I is *not* a maximal ideal.