Homework 4 : Due Monday, February 10

Problem 1: Use the Euclidean Algorithm to show that $\overline{153} \in U(\mathbb{Z}/385\mathbb{Z})$ and to explicitly find its inverse.

Problem 2:
a. Write out the Cayley Table of $U(\mathbb{Z}/18\mathbb{Z})$.
b. Compute (with explanation) the order of $\overline{11}$ in $U(\mathbb{Z}/18\mathbb{Z})$.

Problem 3: Let G be group and let $a, g \in G$. The element gag^{-1} is called a *conjugate* of a.
a. Show that $(gag^{-1})^n = ga^ng^{-1}$ for all $n \in \mathbb{Z}$. You should start by giving a careful inductive argument for $n \in \mathbb{N}$.
b. Show that $|gag^{-1}| = |a|$. Thus, every conjugate of a has the same order as a.
c. Show that $|ab| = |ba|$ for all $a, b \in G$.

Problem 4: Let G be a group with even order. Show that G has an element of order 2.
Hint: You need to show that there is a nonidentity elements which is its own inverse. Think about taking the inverse of each element and see what happens.

Problem 5: Suppose that $n \geq 3$. Let $\sigma \in S_n$ with $\sigma \neq id$. Show that there exists $\tau \in S_n$ such that $\sigma\tau \neq \tau\sigma$.
Hint: I strongly recommend that you avoid cycle notation and just work with σ as a function. Since $\sigma \neq id$, start by fixing an i with $\sigma(i) \neq i$. Now build τ.

Problem 6: Let $n \in \mathbb{N}^+$.
a. Given $k \in \mathbb{N}$ with $2 \leq k \leq n$, find a formula for the number of k-cycles in S_n and explain why it is correct. (Remember that $(1\ 2\ 3) = (2\ 3\ 1)$ so don't count it twice.)
b. Find a formula for the number of permutation in S_n which are the product of two disjoint 2-cycles and explain why it is correct.
c. In the special case of $n = 5$, calculate the number of permutations of S_5 of each cycle type (so you should explicitly calculate the number of 4-cycles, the number of permutations which are the product of a 3-cycle and 2-cycle which are disjoint, etc.). Notice that all of your answers divide $|S_5| = 120$. This is not an accident, as we will see later.