Homework 1 : Due Monday, January 27

Problem 1: Let $a, b, c \in \mathbb{Z}$. Suppose that $a \mid b$ and $a \nmid c$. Show that $a \nmid (b + c)$.

Problem 2: Use induction to show that $6 \mid (2n^3 + 3n^2 + n)$ for all $n \in \mathbb{N}$.

Problem 3: Define a sequence recursively as follows. Let $a_0 = 6$, let $a_1 = 33$, and let $a_n = 7a_{n-1} - 2a_{n-2}$ for all $n \geq 2$. Use strong induction to show that $3 \mid a_n$ for all $n \in \mathbb{N}$. Be sure to state your inductive hypothesis carefully!

Problem 4: Show that for all $a \in \mathbb{Z}$, either there exists $k \in \mathbb{Z}$ with $a^2 = 4k$ or there exists $k \in \mathbb{Z}$ with $a^2 = 4k + 1$.
Hint: Start by using division with remainder on a.

Problem 5: Use the Euclidean Algorithm to find the greatest common divisor of the following pairs of numbers a and b. Furthermore, once you find the greatest common divisor d, find $m, n \in \mathbb{Z}$ such that $am + bn = d$.

- $a = 234$ and $b = 165$.
- $a = 562$ and $b = 471$.