Homework 1: Due Friday, September 6

Problem 1: Let $a, b, c \in \mathbb{Z}$. Suppose that $a \mid b$ and $a \nmid c$. Show that $a \nmid (b + c)$.

Problem 2: Use induction to show that $6 \mid (2n^3 + 3n^2 + n)$ for all $n \in \mathbb{N}$.

Problem 3: Define a sequence recursively as follows. Let $a_0 = 6$, let $a_1 = 33$, and let $a_n = 7a_{n-1} - 2a_{n-2}$ for all $n \geq 2$. Use strong induction to show that $3 \mid a_n$ for all $n \in \mathbb{N}$. Be sure to state your inductive hypothesis carefully!

Problem 4: Show that $Div(a) = Div(-a)$ for all $a \in \mathbb{Z}$.

Problem 5: Show that for all $a \in \mathbb{Z}$, either there exists $k \in \mathbb{Z}$ with $a^2 = 3k$ or there exists $k \in \mathbb{Z}$ with $a^2 = 3k + 1$.
Hint: Start by performing division with remainder on a.

Problem 6: Use the Euclidean Algorithm to find the greatest common divisor of the following pairs of numbers a and b. Furthermore, once you find the greatest common divisor m, find $k, \ell \in \mathbb{Z}$ such that $ka + \ell b = m$.

- $a = 234$ and $b = 165$.
- $a = 562$ and $b = 471$.

Problem 7: Find, with proof, all $n \in \mathbb{Z}$ such that $\gcd(n, n + 2) = 2$.

Problem 8: Let $a, b, c \in \mathbb{Z}$. Suppose that $a \mid c$, that $b \mid c$, and that $\gcd(a, b) = 1$. Using only the material through Section 2.4 (so without using any properties of prime factorizations), show that $ab \mid c$.