Homework 8 : Due Wednesday, September 15

Problem 1: Use the Euclidean Algorithm to show that $\overline{153} \in U(\mathbb{Z}/385\mathbb{Z})$ and to explicitly find its inverse.

Problem 2:
a. Write out the Cayley Table of $U(\mathbb{Z}/18\mathbb{Z})$.
b. Compute the order of $\overline{11}$ in $U(\mathbb{Z}/18\mathbb{Z})$.

Problem 3: Suppose that $n \in \mathbb{N}$ with $n \equiv_4 3$. Show that n is not the sum of two squares, i.e. there does not exist $a, b \in \mathbb{Z}$ with $a^2 + b^2 = n$.

Problem 4: Let $n \in \mathbb{N}^+$ and let $a, b \in \mathbb{Z}$. Show that the following are equivalent:

- There exists $x \in \mathbb{Z}$ with $ax \equiv_n b$.
- $\gcd(a, n)$ divides b.

Problem 5: Suppose that $p, k \in \mathbb{N}^+$ and that p is prime. Show that $\varphi(p^k) = p^k - p^{k-1}$.