Homework 5 : Due Wednesday, September 8

Problem 1: Let f_n be the n^{th} Fibonacci number as defined in Problem 2 on Homework 3. Show that $\gcd(f_n, f_{n+1}) = 1$ for all $n \in \mathbb{N}^+$.

Problem 2: Let $a, b, c \in \mathbb{Z}$ with $a > 0$. Show that $\gcd(ab, ac) = a \cdot \gcd(b, c)$.

Problem 3: Let $a, b, c \in \mathbb{Z}$. Show that the following are equivalent:

- $\gcd(ab, c) = 1$
- $\gcd(a, c) = 1$ and $\gcd(b, c) = 1$

Problem 4: Let $a, b \in \mathbb{N}^+$ and let $d = \gcd(a, b)$. Since d is a common divisor of a and b, we may fix $k, \ell \in \mathbb{N}$ with $a = kd$ and $b = \ell d$. Let $m = k\ell d$.
a. Show that $a \mid m$, $b \mid m$, and $dm = ab$.
b. Show that $\gcd(k, \ell) = 1$.
c. Suppose that $n \in \mathbb{Z}$ is such that $a \mid n$ and $b \mid n$. Show that $m \mid n$.
Because of parts a and c above, the number m is called the *least common multiple* of a and b and is written as $lcm(a, b)$. Since $dm = ab$ from part a, it follows that $\gcd(a, b) \cdot \text{lcm}(a, b) = ab$.