Homework 3 : Due Friday, September 3

Problem 1: Find a formula for

$$\sum_{k=1}^{n}(-1)^{k-1}(2k-1) = 1 - 3 + 5 - 7 + 9 - \cdots + (-1)^{n-1}(2n-1)$$

and prove that your formula is correct.

Problem 2: Let f_n be the the sequence of Fibonacci Numbers, i.e. define $f_1 = 1$, $f_2 = 1$, and $f_n = f_{n-1} + f_{n-2}$ for all $n \geq 3$. Show that

$$f_{n+1}f_{n-1} = f_n^2 + (-1)^n$$

for all $n \geq 2$.

Problem 3: Let $x \in \mathbb{R}$ with $x \geq 0$.
a. Show that $(1+x)^n \geq 1 + nx$ for all $n \in \mathbb{N}^+$ using the Binomial Theorem.
b. Show that $(1+x)^n \geq 1+nx$ for all $n \in \mathbb{N}^+$ using a direct inductive argument (i.e. do not use the Binomial Theorem).
c. Show that

$$\sqrt[n]{2} \leq 1 + \frac{1}{n}$$

for all $n \in \mathbb{N}^+$.

Problem 4: Show that $6 \mid (2n^3 + 3n^2 + n)$ for all $n \in \mathbb{N}$.