Homework 28 : Due Monday, November 29

Problem 1: Either prove or give a counterexample: If F is a field and $f(x), g(x) \in F[x]$ have the property that $f(a) \mid g(a)$ in F for all $a \in F$, then $f(x) \mid g(x)$ in $F[x]$.

Problem 2:
a. Find, with proof, all irreducible polynomials in $\mathbb{Z}/2\mathbb{Z}[x]$ of degree 2 or 3.
b. Show that $x^5 + x^2 + \overline{1} \in \mathbb{Z}/2\mathbb{Z}[x]$ is irreducible.

Problem 3: Let $p \in \mathbb{N}^+$ be prime. Show that there exists an irreducible polynomial in $\mathbb{Z}/p\mathbb{Z}[x]$ of degree 2.
Hint: You don't have to explicitly build one for each prime p. You just need to show one exists.

Problem 4: Determine whether the following polynomials are irreducible in $\mathbb{Q}[x]$.
a. $x^4 - 5x^3 + 3x - 2$
b. $x^4 - 2x^3 + 2x^2 + x + 4$