Homework 27 : Due Monday, November 22

Problem 1: Working in $\mathbb{Z}[i]$, perform the Euclidean algorithm to find *all* greatest common divisors of $11-2i$ and $-1+7i$.

Problem 2: Consider the subset of R given by $R = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$. It is straightforward to check that R is a subring of $\mathbb{R}$.
a. Show that $1 + \sqrt{2}$ is a unit in R.
b. Show that R has infinitely many units.

Problem 3:
a. Show that 5 is not irreducible in $\mathbb{Z}[i]$ (of course, 5 is irreducible in $\mathbb{Z}$).
b. Show that 3 is irreducible in $\mathbb{Z}[i]$. *Hint:* Think about norms.

Problem 4: Let $p \in \mathbb{N}^+$ be prime. Consider the polynomial $f(x) = x^p - x$ in $\mathbb{Z}/p\mathbb{Z}[x]$. How many roots does $f(x)$ have in $\mathbb{Z}/p\mathbb{Z}$? Explain.

Problem 5: Let F be a field. Show that there are infinitely many irreducible polynomials in $F[x]$.