Homework 17 : Due Wednesday, October 13

Problem 1: Determine, with proof, whether the following pairs of groups are isomorphic.
a. A_6 and S_5.
b. $\mathbb{Z}/84\mathbb{Z}$ and $\mathbb{Z}/6\mathbb{Z} \times \mathbb{Z}/14\mathbb{Z}$.
c. $U(\mathbb{Z}/18\mathbb{Z})$ and $\mathbb{Z}/6\mathbb{Z}$.
d. S_4 and $\mathbb{Z}/6\mathbb{Z} \times U(\mathbb{Z}/5\mathbb{Z})$.
e. A_4 and D_6.
f. $S_3 \times \mathbb{Z}/2\mathbb{Z}$ and A_4.
g. $D_4/Z(D_4)$ and $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$.
h. $U(\mathbb{Z}/5\mathbb{Z})$ and $U(\mathbb{Z}/10\mathbb{Z})$.
Hint: Most of these can be done without explicitly building isomorphisms or explicitly ruling out each possibility. Use the theory we have developed.

Problem 2: Let G and H be groups. Show that $G \times H \cong H \times G$.

Problem 3: Consider the group $G = \mathbb{R}\backslash\{-1\}$ with operation $a * b = a + b + ab$ from Homework 2. Let H be the group $\mathbb{R}\backslash\{0\}$ with operation equal to the usual multiplication. Show that $G \cong H$.

Problem 4: Suppose that $G \cong H$ and that G has a subgroup of order m. Show that H has a subgroup of order m.