# Homework 13 : Due Monday, October 4

**Problem 1:** Let $G$ be a group. Suppose that $H$ and $K$ are finite subgroups of $G$ such that $|H|$ and $|K|$ are relatively prime. Show that $H \cap K = \{e\}$.

**Problem 2:** Consider $\mathbb{Q}$ as group under the operation of addition. Notice that $\mathbb{Z}$ is a subgroup of $\mathbb{Q}$. Show that $[\mathbb{Q} : \mathbb{Z}] = \infty$.

**Problem 3:**
a. Let $G$ be a group with the property that $a^2 = e$ for all $a \in G$. Show that $G$ is abelian.
b. Show that every group of order 4 is abelian.
*Note:* Recall that $U(\mathbb{Z}/8\mathbb{Z})$ is an example of abelian group of order 4 which is not cyclic. Since 2, 3, and 5 are prime, it follows that the smallest possible order of a nonabelian group is 6. Indeed, $S_3$ is an example of such a group.

**Problem 4:** Let $p, k \in \mathbb{N}^+$ with $p$ prime. Suppose that $G$ is a group with $|G| = p^k$. Show that $G$ has an element of order $p$.

**Problem 5:** Let $G$ be a group. Suppose that $H$ and $K$ are both subgroups of $G$. Define a relation on $G$ by letting $a \sim b$ mean that there exists $h \in H$ and $k \in K$ with $b = hak$.
a. Show that $\sim$ is an equivalence relation on $G$.
b. The equivalence classes of $\sim$ are called *double cosets*. Find the double cosets in the case where $G = A_4$ and $H = K = \langle (1\ 2\ 3) \rangle$.