Homework 12 : Due Friday, October 1

Problem 1: This problem gives another interpretation of D_n as a subgroup of $GL_2(\mathbb{R})$ (in fact, a subgroup of $O(2,\mathbb{R})$) by thinking of rotation and flips as linear transformations from $\mathbb{R}^2$ to $\mathbb{R}^2$.
a. Let $\alpha, \beta \in \mathbb{R}$. Show that

$$\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} = \begin{pmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) \\ \sin(\alpha+\beta) & \cos(\alpha+\beta) \end{pmatrix}$$

b. Let $n \geq 3$. Let

$$R = \begin{pmatrix} \cos(2\pi/n) & -\sin(2\pi/n) \\ \sin(2\pi/n) & \cos(2\pi/n) \end{pmatrix} \qquad\qquad S = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Show that $|R| = n$, $|S| = 2$, and $SR = R^{-1}S$.

Problem 2: Compute the left cosets of the subgroup H of the given group G in each of the following cases (make sure you completely determine H first!). For the nonabelian groups G in parts c,d,e, also compute the right cosets of H in G.
a. $G = \mathbb{Z}/12\mathbb{Z}$ and $H = \langle \overline{4} \rangle$.
b. $G = U(\mathbb{Z}/18\mathbb{Z})$ and $H = \langle \overline{17} \rangle$ (you computed the Cayley table of $U(\mathbb{Z}/18\mathbb{Z})$ in Homework 8).
c. $G = D_4$ and $H = \langle r^2 s \rangle$.
d. $G = A_4$ and $H = \langle (1\ 2\ 3) \rangle$.
e. $G = D_n$ and $H = \langle r \rangle$.
Hint: Save as much work as you can by using the general fact that you are working with equivalence classes of a certain equivalence relation, and you know that the equivalence classes partition G.

Problem 3: Let H be a subgroup of G and let $a \in G$. Show that if $aH = Hb$ for some $b \in G$, then $aH = Ha$. In other words, if the left coset aH equals *some* right coset of H in G, then it must equal the right coset Ha.
Hint: Again, use the general theory of equivalence relations to simplify your life.

Problem 4: Let G be a group and let H and K be subgroups of G. Let $a \in G$. Show that the two sets $aH \cap aK$ and $a(H \cap K)$ are equal. Thus, the left cosets of the subgroup $H \cap K$ are obtained by intersecting the corresponding left cosets of H and K individually.