

Homework 11 : Due Wednesday, September 29

Problem 1: Show that both A_2 and A_3 are abelian, but A_n is nonabelian whenever $n \geq 4$.

Problem 2: Let $n \geq 3$. Show that the set of 3-cycles generates A_n .

Problem 3: Suppose that $\sigma \in A_n$ and $|\sigma| = 2$. Show that there exists $\tau \in S_n$ with $|\tau| = 4$ and $\tau^2 = \sigma$.

Problem 4: Let $n \geq 3$. Working in D_n , determine $|r^k s^\ell|$ for each $k, \ell \in \mathbb{N}$ with $0 \leq k \leq n-1$ and $0 \leq \ell \leq 1$.

Problem 5: Let $n \geq 3$.

a. Show that if $a \in D_n$ and $a \in \langle r \rangle$, then $sa = a^{-1}s$.

b. Show that if $a \in D_n$ but $a \notin \langle r \rangle$, then $ra = ar^{-1}$.

c. Find $Z(D_n)$. Your answer will depend on whether n is even or odd.