Exam 2: Take-Home
Due Friday, November 5 by 5:00pm

- You may consult the course notes, your own notes from class, old homework and their solutions.
- You man **not** consult any outside sources (books, online materials, etc.) and you may not discuss the problems with anyone.

1. (3 points each) Determine whether the following groups are isomorphic. Justify your answers.
a. $\mathbb{Z}/14\mathbb{Z} \times \mathbb{Z}/15\mathbb{Z}$ and $\mathbb{Z}/210\mathbb{Z}$.
b. D_{36} and $S_3 \times A_4$.

2. (6 points each) For each of the following, either prove or give a counterexample (with justification).
a. If H is a normal subgroup of G, and both H and G/H are cyclic, then G is abelian.
b. Every group of order 100 has an element of order 4.
c. A group of order 30 has at most 7 subgroups of order 5.

3. (8 points) Suppose that H and K are subgroups of G. Suppose that

- $G = \{hk : h \in H, k \in K\}$
- $hk = kh$ for all $h \in H$ and $k \in K$

Show that H and K are normal subgroups of G.

4. (8 points) Let G be a finite group. Suppose that H is a normal subgroup of G and that G/H has an element of order $n \in \mathbb{N}^+$. Show that G has an element of order n.

5. (10 points) Suppose that $|G|$ is odd. Show that if $a \neq e$, then a and a^{-1} are not conjugates in G.