Homework 5 : Due Wednesday, September 9

Extra Problem: Let $a, b \in \mathbb{N}$, and let $d = gcd(a, b)$. Since d is a common divisor of a and b, we may fix $k_a, k_b \in \mathbb{N}$ with $a = k_a \cdot d$ and $b = k_b \cdot d$.

a. Let $m = \frac{a \cdot b}{d}$. Show that m is a natural number, that $a \mid m$, and that $b \mid m$.
b. Show that k_a and k_b are relatively prime.
c. Suppose that $n \in \mathbb{N}$ is such that $a \mid n$ and $b \mid n$. Show that $m \mid n$.

Because of parts b and c above, the number m is called the *least common multiple* of a and b and is written as $lcm(a, b)$. Notice that $d \cdot m = a \cdot b$ from the definition of m, i.e. that $gcd(a, b) \cdot lcm(a, b) = a \cdot b$.

Hint for c: Problem 27 and part b are very useful.