Homework 27 : Due Friday, December 4

Problem 1: Chapter 15: #24abcd

Problem 2: Let $R = \mathbb{Z}[x]$. Show that ideal

$$I = \langle 2, x \rangle = \{f(x) \cdot 2 + g(x) \cdot x : f(x), g(x) \in \mathbb{Z}[x]\}$$

is not a principal ideal of R.

Problem 3: Suppose that R is a commutative ring, that P is prime ideal of R, and that I and J are ideals of R. Suppose that $I \cap J \subseteq P$. Show that either $I \subseteq P$ or $J \subseteq P$ (or both).

Problem 4: Let F be a field and let $R = F[x]$.
a. Show that $I = \{0\}$ is a prime ideal of R which is not maximal.
b. Show that if I is a prime ideal of R other than $\{0\}$, then I is maximal.

Problem 5: Show that there exist fields of order 27 and 49.