

## Homework 21 : Due Monday, November 9

**Problem 1:** Chapter 12, #2ab

**Problem 2:** Chapter 12, #3

**Problem 3:** Chapter 12, #5ab

*Hint:* Use a bit of theory to help with the computations. You know the orbits partition  $X$ , and you know that the size of any orbit divides  $|G|$ . Once you know  $\mathcal{O}_x$ , you can use the Orbit-Stabilizer Theorem to determine  $|G_x|$ .

**Problem 4:** Let  $G = \mathbb{R}$  (under addition) and let  $X = \mathbb{R}^2$ . Define a function from  $G \times X$  to  $X$  by  $a * (x, y) = (x + ay, y)$ .

- Show that  $*$  is an action of  $G$  on  $X$ .
- Describe the orbits of the action geometrically.

**Problem 5:** Let  $G = S_3$  and let

$$X = \{1, 2, 3\} \times \{1, 2, 3\} = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

Define a function from  $G \times X$  to  $X$  by  $\sigma * (x, y) = (\sigma(x), \sigma(y))$ .

- Show that  $*$  is an action of  $G$  on  $X$ .
- Find the orbits of the action.