# Homework 7: Due Friday, October 19

**Problem 1:** With our work on rearrangements of series, you might be concerned about rearrangements of sequences. Suppose then that $\langle a_n \rangle$ is a sequence that converges to $\ell \in \mathbb{R}$. Show that if $\langle b_n \rangle$ is a rearrangement of $\langle a_n \rangle$, i.e. if there exists a bijection $f : \mathbb{N}^+ \to \mathbb{N}^+$ such that $b_{f(n)} = a_n$ for all $n \in \mathbb{N}^+$, then $\langle b_n \rangle$ converges to $\ell$.

**Problem 2:** Either prove or give a counterexample for each of the following:
a. For all $A, B \subseteq \mathbb{R}$, we have $\text{int}(A \cup B) = \text{int}(A) \cup \text{int}(B)$.
b. For all $A, B \subseteq \mathbb{R}$, we have $\text{int}(A \cap B) = \text{int}(A) \cap \text{int}(B)$.

**Problem 3:** Let $A \subseteq \mathbb{R}$. Given $b \in \mathbb{R}$, we say that $b$ is an *isolated point* of $A$ if there exists $\delta > 0$ such that $V_\delta(b) \cap A = \{b\}$. Notice that if $b$ is an isolated point of $A$, then $b \in A$.
a. Show that if $b$ is an isolated point of $A$, then $b \in \text{cl}(A)$, but $b$ is not a limit point of $A$.
b. Show that for any set $A \subseteq \mathbb{R}$, we have

$$\text{cl}(A) = \{b \in \mathbb{R} : b \text{ is an isolated point of } A\} \cup \{b \in \mathbb{R} : b \text{ is a limit point of } A\}.$$

**Problem 4:** Let $A \subseteq \mathbb{R}$ be nonempty and bounded above.
a. Show that $\sup A \in \text{cl}(A)$.
b. Show that if $A$ is open, then $\sup A \notin A$.

**Problem 5:** Let $\langle a_n \rangle$ be a sequence. Show that $\{b \in \mathbb{R} : b \text{ is a cluster point of } \langle a_n \rangle\}$ is closed.