Problem Set 8

Problem 1: How many zeros does 10000! end with?

Problem 2: Let R be the region consisting of the points (x, y) of the cartesian plane satisfying both $|x| - |y| \leq 1$ and $|y| \leq 1$. Sketch the region R and find its area.

Problem 3: Let $n \in \mathbb{Z}$. Show that $\gcd(n^2 + 1, (n + 1)^2 + 1)$ is either 1 or 5.

***Problem 4:** Find all positive integers n such that $n = d(n)^2$, where $d(n)$ equals the number of positive divisors of n (for example, $d(9) = 3$).

***Problem 5:** Prove that the expression

$$\frac{\gcd(m, n)}{n}\binom{n}{m}$$

is an integer whenever $n \geq m \geq 1$.

***Problem 6:** Given $n \in \mathbb{N}^+$, let $[n] = \{1, 2, 3, \ldots, n\}$.
a. For which values of n is it possible to express $[n]$ as the union of two non-empty disjoint subsets so that the elements in the two subsets have equal sum?
b. For which values of n is it possible to express $[n]$ as the union of three non-empty disjoint subsets so that the elements in the three subsets have equal sum?

***Problem 7:** Let d be a real number. For each integer $m \geq 0$, define a sequence $\{a_m(j)\}$ by the condition

$$a_m(0) = \frac{d}{2^m} \qquad\qquad a_m(j + 1) = (a_m(j))^2 + 2a_m(j)$$

Evaluate $\lim_{n\to\infty} a_n(n)$.