Problem Set 7

Problem 1: Show that if m, n are positive integers, then

$$\frac{1}{\sqrt[n]{m}} + \frac{1}{\sqrt[m]{n}} > 1$$

Problem 2: Suppose that $f\colon \mathbb{R} \to \mathbb{R}$ has the property that

$$|f(a) - f(b)| \leq (a-b)^2$$

for all $a, b \in \mathbb{R}$. Show that f is a constant function.

Problem 3: Find a nonzero polynomial $P(x, y)$ such that $P(\lfloor a \rfloor, \lfloor 2a \rfloor) = 0$ for all real numbers a. Here, $\lfloor v \rfloor$ is the greatest integer less than or equal to v.

***Problem 4:** Show that if d is a positive integer, then at least one of the numbers $2d-1$, $5d-1$, or $13d-1$ is not a perfect square.

***Problem 5:** Let $(x_n)_{n\geq 0}$ be a sequence of nonzero real numbers such that

$$x_n^2 - x_{n-1}x_{n+1} = 1$$

for $n \geq 1$. Prove there exists a real number a scuh that $x_{n+1} = ax_n - x_{n-1}$ for all $n \geq 1$.

***Problem 6:** Find all positive integers that are within 250 of exactly 15 perfect squares.

***Problem 7:** For which real numbers c is there a straight line that intersects the curve

$$y = x^4 + 9x^3 + cx^2 + 9x + 4$$

in four distinct points?