Problem Set 6

Problem 1: Define a sequence a_n by letting $a_0 = c$ and

$$a_{n+1} = \frac{a_n}{1 + na_n}$$

Determine a_{2011} in terms of c.

Problem 2: Place a knight on each square of a 7×7 chessboard. Is it possible for each knight to simultaneously make a legal move?

***Problem 3:** The numbers 1 through 10 are placed around a circle. Prove that you can find three consecutive numbers around the circle whose sum is at least 18.

***Problem 4:** Prove that there exist infinitely many integers n such that n, $n+1$, $n+2$ are each the sum of two squares of integers. Example: $0 = 0^2 + 0^2$, $1 = 0^2 + 1^2$, $2 = 1^2 + 1^2$.

***Problem 5:** Suppose that $p(x)$ is a polynomial of degree n which satisfies $p(k) = 2^k$ whenever $0 \leq k \leq n$. Determine $p(n+1)$.

***Problem 6:** Two real numbers x and y are chosen at random in the interval $(0, 1)$ with respect to the uniform distribution. What is the probability that the closest integer to x/y is even? Express the answer in the form $r + s\pi$ where r and s are rational numbers.