Problem Set 1

Problem 1: Show that given any five integers, one can always find three whose sum is divisible by 3.

Problem 2: A group of n people play a round-robin tournament. Each game ends in either a win or a loss. Show that it is possible to label the players $P_1, P_2, \ldots, P_n$ in such a way that P_1 defeated P_2, P_2 defeated P_3, $\ldots$, P_{n-1} defeated P_n.

Problem 3: Let $f_0(x) = \frac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x))$. Evaluate $f_{2011}(2011)$.

Problem 4: Let n be a positive integer. Prove that 2^{n-1} divides $n!$ if and only if n is a power of 2.

***Problem 5:** Suppose that every point in the plane is colored either red, white, or blue. Show that there exists two points in the plane which are exactly 1 inch apart and have the same color.

***Problem 6:** Prove that there do not exist polynomials $f(x), g(x)$ such that

$$\log x = \frac{f(x)}{g(x)}$$

for all $x > 0$.

***Problem 7:** Determine, with proof, the number of ordered triples (A_1, A_2, A_3) of sets which have the property that

1. $A_1 \cup A_2 \cup A_3 = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
2. $A_1 \cap A_2 \cap A_3 = \emptyset$.

Express the answer in the form $2^a 3^b 5^c 7^d$ where a, b, c, d are nonnegative integers.