Homework 9: Due Wednesday, March 16

Problem 1: Suppose that we have a certain group of 73 students, and that every student in the group is enrolled in either a math class, a computer science class, or a physics class. We also know the following information about the group:

- Number of students enrolled in a math class: 45.

- Number of students enrolled in a computer science class: 33.

- Number of students enrolled in a physics class: 30

- Number of students enrolled in both a math class and a computer science class: 13.

- Number of students enrolled in both a math class and a physics class: 15.

- Number of students enrolled in a math class, a computer science class, and a physics class: 3.

How many of the students are enrolled in both a computer science class and a physics class?

Problem 2: In several cards games (bridge, spades, hearts, etc.) each player receives a 13-card hand from a standard 52-card deck.
a. How many such 13-card hands have at least one card of every suit? What percentage of all possible 13-card hands is this?
b. How many such 13-card hands have all four cards of some rank (e.g. all four queens)?

Problem 3: Consider all 10^{10} many ten digit numbers where you allow leading zeros (so 0018345089 is one possibility). How many such numbers have the property that every odd digit occurs at least once?

Problem 4: Show that if A and B are countable sets, then $A \times B$ is countable.

Problem 5: Show that the set $\mathbb{R} \backslash \mathbb{Q}$ of all irrational numbers is uncountable.

Problem 6:
a. Recall that $\{0, 1\}^*$ is the set of all finite sequences of 0's and 1's (of any finite length). Show that $\{0, 1\}^*$ is countable.
b. Let S be the set of all infinite sequences of 0's and 1's (so an element of S looks like $11000101110\ldots$). Show that S is uncountable.