Homework 6: Due Wednesday, March 2

Problem 1: Using the function $d\colon \mathbb{N}^+ \to \mathbb{N}^+$ from Definition 3.3.4, show that

$$\{n \in \mathbb{N}^+ : d(n) \text{ is odd}\} = \{m^2 : m \in \mathbb{N}^+\}.$$

Problem 2: Show that for all $a \in \mathbb{N}^+$, there exists $k, n \in \mathbb{N}$ with $a = 2^k \cdot n$, and where n is odd.

Problem 3: A number $n \in \mathbb{N}^+$ is called *square-free* if all the primes in the unique prime factorization of n occur only to the first power, i.e. if n can be written as a product of distinct primes. Show that for all $a \in \mathbb{N}^+$, there exists $m, n \in \mathbb{N}^+$ with $a = m^2 \cdot n$, and where n is square-free.

Problem 4: Let A, B, C be sets and let $f\colon A \to B$ and $g\colon B \to C$ be functions. Show that if $g \circ f$ is surjective and g is injective, then f is surjective.

Problem 5: Let $\sigma\colon \mathbb{N}^+ \to \mathbb{N}^+$ be the function defined in Problem 6 on Homework 5, i.e. $\sigma(n)$ is the sum of all positive divisors of n.
a. Is σ injective? Justify your answer.
b. Is σ surjective? Justify your answer.

Problem 6: Using the digits 1 through 9 only (so exclude 0), how many 13-digit numbers are there in which no two consecutive digits are the same? Explain your reasoning.