Homework 12: Due Wednesday, April 20

Problem 1: Suppose $p, n \in \mathbb{N}^+$ and that p is prime.
a. Show that if $p \nmid n$, then $\varphi(pn) = (p-1) \cdot \varphi(n)$.
b. Show that if $p \mid n$, then $\varphi(pn) = p \cdot \varphi(n)$.
c. Show that $\varphi(2n) = \varphi(n)$ if n is odd and that $\varphi(2n) = 2 \cdot \varphi(n)$ if n is even.

Problem 2: Show that $\varphi(n)$ is even whenever $n > 2$.

Problem 3: Find, with full explanation, all $n \in \mathbb{N}^+$ with $\varphi(n) = 4$.

Problem 4: Find, with full explanation, the remainder when dividing 3^{846} by 308.

Problem 5: Find, with full explanation, all $x \in \mathbb{Z}$ such that both $8x \equiv 3 \pmod{13}$ and $3x \equiv 2 \pmod{20}$.
Hint: Solve each congruence in isolation first, and then use the Sun-Tzu Remainder Theorem.

Problem 6: Show that $a^{91} \equiv a^7 \pmod{91}$ for all $a \in \mathbb{Z}$.
Hint: Use the fact that $91 = 7 \cdot 13$ is the prime factorization of 91.