Homework 11: Due Friday, March 19

Exercises

Exercise 1: We proved that $m \leq 3n - 6$ for all *connected* planar graphs with $n \geq 3$ vertices and m edges. Explain why this is true for all planar graphs (i.e. if we omit the word connected).

Exercise 2: Let G be a planar graph with no triangles.
a. Show that G has a vertex v with $d(v) \leq 3$.
b. Without appealing to the general Four Color Theorem, show that $\chi(G) \leq 4$.

Problems

Problem 1: Show that a finite tree has at most one perfect matching.
Hint: Given two perfect matchings, think about the symmetric difference.

Problem 2:
a. Show that if you remove any two edges from K_6, then the resulting graph is not planar.
b. Show that it is possible to remove three edges from K_6 so that that resulting graph is planar.
c. Show that it is possible to remove three edges from K_6 so that that resulting graph is not planar.

Problem 3: Let G be a graph with $n \geq 11$ vertices. Show that at most one of G or $\overline{G}$ is planar.

Problem 4: Suppose that you color the edges of K_n using 2 colors. Show that there exists a spanning tree T of K_n such that all edges of T have the same color.