Homework 14: Due Monday, April 27

Problem 1: Show that if $p \in \mathbb{N}^+$ is prime, then $(p-2)! \equiv 1 \pmod{p}$.

Problem 2: Let $m \in \mathbb{Z}$ with $m \equiv 3 \pmod{4}$. Show that there does not exist $a, b \in \mathbb{Z}$ with $m = a^2 + b^2$.
Hint: Start by consider the possible values of a^2 modulo 4.

Problem 3: Show that $19 \nmid 4n^2 + 4$ for all $n \in \mathbb{Z}$.

Problem 4: Find, with full explanation, the remainder when dividing 3^{846} by 308.

Problem 5: Show that $\varphi(n)$ is even whenever $n > 2$.

Problem 6: Recall that Wilson's Theorem says that $(p-1)! \equiv -1 \pmod{p}$ whenever p is prime. Notice that $(4-1)! = 6$, so $(4-1)! \equiv 2 \pmod{4}$. Show that if $n \in \mathbb{N}$ is composite and $n > 4$, then $(n-1)! \equiv 0 \pmod{n}$.
Note: In particular, it follows that $(n-1)! \not\equiv -1 \pmod{n}$ whenever n is composite, giving a converse to Wilson's Theorem.