# Homework 8 : Due Wednesday, March 5

**Problem 1:** Let $m, n, k \in \mathbb{N}^+$ with $k \le m$ and $k \le n$. Calculate the number of sequences of zeros and ones of length $m + n$ which have both of the following properties:

- There are exactly $m$ zeros and $n$ ones.

- There are exactly $k$ runs of ones.

Thus, you should no longer assume that the sequence starts with a one and ends with a zero.

**Problem 2:** In several cards games (bridge, spades, hearts, etc.) each player receives a 13-card hand from a standard 52-card deck.
a. How many such 13-card hands have at least one card of every suit? What percentage of all possible 13-card hands is this?
b. How many such 13-card hands have all four cards of some rank (e.g. all four queens)?

**Problem 3:** Consider all $10^{10}$ many ten digit numbers where you allow leading zeros (so 0018345089 is one possibility). How many such numbers have the property that every odd digit occurs at least once?

**Problem 4:** Determine the number of solutions to

$$x_1 + x_2 + x_3 + x_4 = 17$$

where each $x_i \in \mathbb{N}$ and each $x_i \le 6$.

**Problem 5:** Given $n$ distinct letters, how many "words" of length $2n$ can you make with both of the following properties:

- Each letter is used exactly twice.

- No two consecutive letters agree.

*Hint:* First count the number of words where each letter is used exactly twice (so just the first condition). Now using Inclusion-Exclusion, count the complement, i.e. those words where each letter is used exactly twice but there exist two consecutive letters that agree. To count the number of "words" in which the two $a$'s are consecutive, think about gluing the two $a$'s together and treating them as one object.