Homework 5 : Due Wednesday, February 19

Note: For each of these counting problems, you must explain your solution. For example, if your answer is a product, describe the sequence of choices you are making and explain where each term comes from. Numerical answers without written justification will receive no credit.

Problem 1: Using the digits 1 through 9 only (so exclude 0), how many 13 digits numbers are there in which no two consecutive digits are the same?

Problem 2: Suppose that you are creating a password using 26 letters, 10 numbers, and 15 special characters. How many such 10-character passwords are possible if they must have exactly 6 letters, 2 numbers, and 2 special characters?

Problem 3: How many ways are there to pick two cards from a standard 52-card deck such that the first card is a spade and the second is not an ace? In this problem, order matters. So if you pick the 3 of spades followed by the 7 of spades, this is different from the 7 of spades followed by the 3 of spades.

Problem 4: Suppose that a lottery draws 6 numbers from $[60] = \{1, 2, \ldots, 60\}$ without replacement and where order drawn doesn't matter. What percentage of possible lottery numbers have 3 evens and 3 odds?

Problem 5: A local pizza place has three different types of crust, five different meats, and seven different (non meat) toppings. For a given pizza, you can pick any crust, at most 2 meats (so 0, 1, or 2 is possible) and at most 3 toppings (so 0, 1, 2, or 3 is possible). How many pizzas are possible?

Problem 6: How many 6-letter "words" contain one of the letters A, B, C, D three times and each of the others once?

Problem 7: In class, we talked about the number of paths starting at $(0, 0)$ and ending at (m, n) where each step was either one step north or step east. How many such paths are there from $(0, 0)$ to $(12, 9)$ which do not go through the point $(5, 4)$? Think of needing to avoid that intersection because of construction.

Problem 8: How many 5-card poker hands have at least one card of every suit?

Problem 9: Snow White and the seven dwarfs (Bashful, Doc, Dopey, Grumpy, Happy, Sleepy and Sneezy) plan to eat dinner at a long rectangular table with four seats on each of two sides, and no seats on the two ends (so there there are four pairs of two seats opposite each other). Suppose that all seats are considered distinct.
a. Grumpy gets excessively grumpy if he is seated directly across the table from Sneezy. How many ways can the eight be seated so that those two are not across from each other?
b. Doc insists on sitting directly between Dopey and Sleepy to ensure they don't make fools of themselves. How many such seating arrangements are there?
c. How many arrangements satisfy both of the demands of part a and part b?