## Homework 13 : Due Friday, April 18

**Problem 1:** Let $T$ be the unique tree with vertex set $[8]$ whose Prüfer code is $4, 1, 1, 4, 3, 1$. Find the corresponding sequence $a_1, a_2, \ldots, a_7$ and then draw $T$.

**Problem 2:** Count the number of trees with vertex set $[11]$ where all of the following hold:

- $d(5) = 4$
- $d(1) = d(7) = 3$
- $d(4) = d(8) = 2$
- $d(v) = 1$ for all other vertices, i.e. all other vertices are leaves.

**Problem 3:** Using Stirling numbers, count the number of trees with vertex set $[20]$ having exactly 6 leaves.

**Problem 4:** Let $G$ be a connected graph that is not a tree. Show that $G$ has at least 2 spanning trees.

**Problem 5:** Let $G$ be a connected graph with at least 2 vertices. Show that there exist distinct vertices $u$ and $w$ such that both $G - u$ and $G - w$ are connected.
*Hint:* First think about the case where $G$ is a tree.

**Problem 6:** Either prove or find a counterexample: Suppose that $T$ is a minimum weight spanning tree of a connected weighted graph $G$. Let $u$ and $w$ be vertices of $G$. A $u, w$-path in $T$ must have total weight less than or equal to the total weight of each $u, w$-path in $G$.