

## Homework 1 : Due Monday, January 27

**Problem 1:** Write out the following sets explicitly, and state their cardinalities.

- a.  $\mathcal{P}(\{1, \{2, 3\}\})$
- b.  $\mathcal{P}(\mathcal{P}(\{2\}))$
- c.  $\mathcal{P}(\emptyset)$  (be careful!)

**Problem 2:** Give an example of three finite sets  $A_1, A_2, A_3$  such that  $A_1 \cap A_2 \cap A_3 = \emptyset$  but

$$|A_1 \cup A_2 \cup A_3| \neq |A_1| + |A_2| + |A_3|.$$

**Problem 3:** Given two sets  $A$  and  $B$ , we define

$$A \triangle B = \{x : x \text{ is an element of exactly one of } A \text{ or } B\}$$

and we call this set the *symmetric difference* of  $A$  and  $B$ .

- a. Determine  $\{1, 3, 8, 9\} \triangle \{2, 3, 4, 7, 8\}$ .
- b. Determine  $\{1, 2, 3\} \triangle \{1, \{2, 3\}\}$ .
- c. What are the smallest 9 elements of the set  $\{2n : n \in \mathbb{N}\} \triangle \{3n : n \in \mathbb{N}\}$ ?
- d. Make a conjecture about how to write  $\{2n : n \in \mathbb{N}\} \triangle \{3n : n \in \mathbb{N}\}$  as the union of 3 pairwise disjoint sets (no need to prove this conjecture, but do write the 3 sets parametrically).
- e. Show that if  $A$  and  $B$  are finite sets, then  $|A \triangle B| = |A \setminus B| + |B \setminus A|$ .
- f. Show that if  $A$  and  $B$  are finite sets, then  $|A \triangle B| = |A \cup B| - |A \cap B|$ .

*Note:* For parts e and f, appeal to the fundamental counting rules from class or Section 1.5 of the notes.

**Problem 4:** Let  $A$  be a finite set with  $|A| = n$ . Let  $D = \{(a, b) \in A^2 : a = b\}$ .

- a. Write the set  $D$  parametrically.
- b. What is  $|A^2 \setminus D|$ ? Explain.

**Problem 5:** Give careful double containment proofs of each of the following.

- a.  $\{4n : n \in \mathbb{Z}\} = \{4n + 8 : n \in \mathbb{Z}\}$
- b.  $\{\sqrt{5x^4 + 9} : x \in \mathbb{R}\} = \{x \in \mathbb{R} : x \geq 3\}$

*Note:* Given  $x \in \mathbb{R}$  with  $x \geq 0$ , we define  $\sqrt{x}$  to be the unique *nonnegative*  $y \in \mathbb{R}$  with  $y^2 = x$ .