Homework 2 : Due Friday, February 3

Problem 1: Show that for all $n \in \mathbb{N}$, the remainder when you divide n^2 by 4 is always either 0 or 1. *Hint:* Take an arbitrary $n \in \mathbb{N}$ and consider the four possible remainders as four different cases.

Problem 2: Show that $6 \mid (2n^3 + 3n^2 + n)$ for all $n \in \mathbb{N}$.

Problem 3: Show that

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$$

for all $n \in \mathbb{N}^+$.

Problem 4: Let $r \in \mathbb{R}$ with $r \neq 1$. Show that

$$1 + r + r^2 + \cdots + r^n = \frac{r^{n+1} - 1}{r - 1}$$

for all $n \in \mathbb{N}$.

Problem 5: Define a sequence recursively by letting $a_0 = 42$ and letting

$$a_{n+1} = a_n^2 - 3a_n + 14$$

Show that $7 \mid a_n$ for all $n \in \mathbb{N}$.

Problem 6: Show that $2^n > n^2$ for all $n \in \mathbb{N}$ with $n \geq 5$.