

Homework 15 : Due Wednesday, May 9

Problem 1: Let G be a simple planar graph with no triangles. Show that G has a vertex v with $d(v) \leq 3$.

Problem 2: Let G be a simple planar graph with no triangles. Without appealing to the general Four Color Theorem, show that $\chi(G) \leq 4$.

Problem 3:

- Show that if you remove any two edges from K_6 , then the resulting graph is not planar.
- Show that it is possible to remove three edges from K_6 so that that resulting graph is planar.
- Show that it is possible to remove three edges from K_6 so that that resulting graph is not planar.

Problem 4: Let G be a simple graph with $n \geq 11$ vertices. Show that at most one of G or \overline{G} is planar.

Problem 5: Suppose that you color the edges of K_n using 2 colors. Show that there exists a spanning tree T of K_n such that all edges of T have the same color.

Problem 6:

- Suppose that you color the edges of K_{17} using 3 colors. Show that there exists a monochromatic triangle.
- Suppose that you color the edges of K_{66} using 4 colors. Show that there exists a monochromatic triangle.