Homework 13 : Due Friday, April 20

Problem 1: Let T be the unique tree with vertex set $[8]$ whose Prüfer code is $5, 2, 2, 5, 1, 2$. Find the corresponding sequence $a_1, a_2, \ldots, a_7$ and then draw T.

Problem 2: Count the number of trees with vertex set $[11]$ where all of the following hold:

- $d(5) = 4$
- $d(1) = d(7) = 3$
- $d(4) = d(8) = 2$
- $d(v) = 1$ for all other vertices, i.e. all other vertices are leaves.

Problem 3: Using Stirling numbers, count the number of trees with vertex set $[20]$ having exactly 6 leaves.
Hint: Think about surjections.

Problem 4: Let G be a connected graph with at least 2 vertices. Show that there exist distinct vertices u and w such that both $G - u$ and $G - w$ are connected.
Hint: Think about spanning trees.

Problem 5: Let G be a simple connected graph that is not a tree. Show that G has at least 2 spanning trees.

Problem 6: Either prove or find a counterexample: Suppose that T is a minimum weight spanning tree of a connected weighted graph G. Let u and w be vertices of G. The unique u, w-path in T always has total weight less than or equal to the total weight of every u, w-path in G.