Homework 15: Due Wednesday, November 23

Problem 1: Let $p_1, p_2, \ldots, p_k$ be the first k primes. Is it always possible to find a number that leaves remainder 1 when divided by each p_i? Explain.

Problem 2: Find, with full explanation, all $x \in \mathbb{Z}$ such that both $8x \equiv 3 \pmod{13}$ and $3x \equiv 2 \pmod{20}$. *Hint:* Solve each equation in isolation first.

Problem 3: Show that $n^{91} \equiv n^7 \pmod{91}$ for all $n \in \mathbb{Z}$.

Problem 4: Suppose that $n \geq 2$ and that n has k distinct odd prime divisors. Show that $2^k \mid \varphi(n)$.

Problem 5:
a. Find, with full explanation, all $n \in \mathbb{N}^+$ with $\varphi(n) = 8$.
b. Show that there are only finitely many $n \in \mathbb{N}^+$ with $\varphi(n) = 30$ (no need to find them).

Problem 6: Let $m > 2$, and let $\{b_1, b_2, \ldots, b_{\varphi(m)}\}$ be a reduced residue system modulo m. Show that $b_1 + b_2 + \cdots + b_{\varphi(m)} \equiv 0 \pmod{m}$.