Homework 1 : Due Friday, September 2

Problem 1: Write out the following sets explicitly, and state their cardinalities.
a. $\mathcal{P}(\{1, 2, 3\})$
b. $\mathcal{P}(\{1, \{2, 3\}\})$
c. $\mathcal{P}(\mathcal{P}(\{2\}))$
d. $\mathcal{P}(\emptyset)$ (be careful!)

Problem 2: Give an example of three finite sets A_1, A_2, A_3 such that $A_1 \cap A_2 \cap A_3 = \emptyset$ but

$$|A_1 \cup A_2 \cup A_3| \neq |A_1| + |A_2| + |A_3|.$$

Problem 3: Given two sets A and B, we define

$$A \triangle B = \{x : x \text{ is an element of exactly one of } A \text{ or } B\},$$

and we call this set the *symmetric difference* of A and B.
a. Determine $\{1, 3, 8, 9\} \triangle \{2, 3, 4, 7, 8\}$.
b. Determine $\{1, 2, 3\} \triangle \{1, \{2, 3\}\}$.
c. What are the smallest 9 elements of the set $\{2n : n \in \mathbb{N}\} \triangle \{3n : n \in \mathbb{N}\}$?
d. Make a conjecture about how to write $\{2n : n \in \mathbb{N}\} \triangle \{3n : n \in \mathbb{N}\}$ as the union of 3 pairwise disjoint sets (no need to prove this conjecture, but do write the 3 sets parametrically).
e. Show that if A and B are finite sets, then $|A \triangle B| = |A \backslash B| + |B \backslash A|$.
f. Show that if A and B are finite sets, then $|A \triangle B| = |A \cup B| - |A \cap B|$.
Note: For parts e and f, appeal to the fundamental counting rules from class or Section 1.2 of the notes.

Problem 4: Let A be a finite set with $|A| = n$. Let $D = \{(a, b) \in A^2 : a = b\}$.
a. Write the set D parametrically.
b. What is $|A^2 \backslash D|$? Explain.

Problem 5: Give a careful proof showing that $\{6n^2 + 14 : n \in \mathbb{Z}\} \subseteq \{2n - 8 : n \in \mathbb{Z}\}$.

Problem 6: Give a careful double containment proof showing that $\{\sqrt{5x^2 + 9} : x \in \mathbb{R}\} = \{x \in \mathbb{R} : x \geq 3\}$.
Recall: Given $x \in \mathbb{R}$ with $x \geq 0$, we define $\sqrt{x}$ to be the unique *nonnegative* $y \in \mathbb{R}$ with $y^2 = x$.