Homework 14 : Due Wednesday, December 7

Problem 1: Prove or find a counterexample: If G is a connected simple graph, then $\chi(G) \leq 1 + a(G)$, where $a(G)$ is the average degree of the vertices in G.

Problem 2:
a. Show that Q_3 is planar.
b. Show that Q_4 is not planar.

Problem 3:
a. Show that if you remove any two edges from K_6, then the resulting graph is not planar.
b. Show that it is possible to remove three edges from K_6 so that that resulting graph is planar.
c. Show that it is possible to remove three edges from K_6 so that that resulting graph is not planar.

Problem 4: Let $k \in \mathbb{N}^+$. A graph G is called k-regular if $d(v) = k$ for all vertices v. Show that for every $m \in \mathbb{N}$ with $m \geq 3$, there exists a simple 4-regular planar graph with $2m$ vertices.

Problem 5: Prove or find a counterexample: Every simple 4-regular planar graph has a triangle.

Problem 6: Suppose that you color the edges of the complete graph K_n using two colors. Show that there exists a spanning tree T of K_n such that all edges of T have the same color.
Hint: See Homework 11.

Problem 7: Ramsey's Theorem is still true when you color the edges using $k \in \mathbb{N}^+$ colors instead of just 2, but of course the required size of n changes. Given $k \in \mathbb{N}^+$, let r_k be the least n such that whenever you color the edges of K_n with k colors, there exist a monochromatic triangle (i.e. a triangle with all edges the same color). Thus, $r_2 = R(3, 3) = 6$. Show that $r_k \leq k(r_{k-1} - 1) + 2$ whenever $k \geq 3$.