Homework 1 : Due Friday, September 2

Problem 1: Let $a, b, c \in \mathbb{Z}$. For each of the following, either prove or find a counterexample.
a. If $ab \mid c$, then $a \mid c$ and $b \mid c$.
b. If $a \mid bc$, then either $a \mid b$ or $a \mid c$.
c. If $a \mid b$ and $a \nmid c$, then $a \nmid (b+c)$.
d. If $ac \mid bc$, then $a \mid b$. Be careful!

Problem 2: Show that $6 \mid (2n^3 + 3n^2 + n)$ for all $n \in \mathbb{N}$.

Problem 3: Define a sequence recursively by letting $a_0 = 39$ and

$$a_{n+1} = a_n^2 - 5a_n + 12$$

Show that $3 \mid a_n$ for all $n \in \mathbb{N}$.

Problem 4: Let $r \in \mathbb{R}$ with $r \neq 1$. Use induction to show that

$$1 + r + r^2 + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}$$

for all $n \in \mathbb{N}$.

Problem 5: Show that

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$$

for all $n \in \mathbb{N}^+$.

Problem 6: Find a formula for

$$\sum_{k=1}^{n} (-1)^{k-1}(2k-1) = 1 - 3 + 5 - 7 + 9 - \cdots + (-1)^{n-1}(2n-1)$$

and prove that your formula is correct for all $n \in \mathbb{N}^+$.