Writing Assignment 8: Due Wednesday, April 17

Problem 1: Let V be a vector space. Suppose that U and W are both subspaces of V. You showed in Writing Assignment 7 that $U \cup W$ might not be a subspace of V. Instead, let

$$U + W = \{\vec{v} \in V : \text{There exists } \vec{u} \in U \text{ and } \vec{w} \in W \text{ with } \vec{v} = \vec{u} + \vec{w}\}.$$

That is, $U + W$ is the set of all vectors in V that can be written as the sum of an element of U and an element of W. Show that $U + W$ is a subspace of V.

Problem 2: Let V be a vector space and let $\vec{u}, \vec{w} \in V$. Show that $\mathsf{Span}(\vec{u} + \vec{w}, \vec{u} - \vec{w}) = \mathsf{Span}(\vec{u}, \vec{w})$.

Problem 3: Let V be a vector space, and let $\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n, \vec{w}_1, \vec{w}_2, \dots, \vec{w}_m \in V$. Assume that $(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n)$ is linearly dependent. Show that $(\vec{u}_1, \vec{u}_2, \dots, \vec{u}_n, \vec{w}_1, \vec{w}_2, \dots, \vec{w}_m)$ is linearly dependent.