

Writing Assignment 6: Due Wednesday, March 13

Problem 1: Suppose that A is an 2×2 idempotent matrix (recall this means that $A^2 = A$).

- Show that the only possible eigenvalues of A are 0 and 1.
- Show that if $A \neq I$, then A is not invertible.
- Show that if $A \neq I$, then 0 must be an eigenvalue of A .

Problem 2: Let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation. Show that $\text{Null}(T) \neq \{\vec{0}\}$ if and only if there exists a basis α of \mathbb{R}^2 such that the second column of $[T]_\alpha$ is the zero vector.

Note: Recall that “if and only if” means that each of the two directions are true, so you need to argue both.