Writing Assignment 2: Due Wednesday, February 7

Problem 1: Let A, B, and C be sets. By giving a careful double containment proof, show that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

Problem 2: Given $a, b \in \mathbb{Z}$, we write $a \mid b$ to mean that there exists $m \in \mathbb{Z}$ with $b = am$. Notice that this the correct way to define "a divides evenly into b" without having to describe division with remainder in the integers.
a. Show that for all $a, b, c \in \mathbb{Z}$, if $a \mid b$ and $a \mid c$, then $a \mid b + c$.
b. Show that for all $a, b, k \in \mathbb{Z}$, if $a \mid b$, then $a \mid kb$.

Problem 3: In Proposition 1.4.9, we showed that every odd integer can be written as the difference of two perfect squares. In this problem, we show that every integer that is evenly divisible by 4 can be written as the difference of two perfect squares. That is, you will prove the following:

$$\text{"For all } a \in \mathbb{Z}, \text{ if } 4 \mid a, \text{ then there exists } b, c \in \mathbb{Z} \text{ with } a = b^2 - c^2\text{"}.$$

However, we will do it in stages:
a. Write down some examples of integers that satisfy the hypothesis, i.e. some examples of integers a for which $4 \mid a$. For each of these, find examples of integers b and c with $a = b^2 - c^2$.
b. If you write down enough examples in part (a), you will hopefully start to see a pattern. If you follow that pattern, what values of b and c do you think will work for $a = 60$? What about $a = 100$?
c. Try to make a guess as to a general pattern. In other words, if $4 \mid a$ and we fix $k \in \mathbb{Z}$ with $a = 4k$, then what do you guess will work for b and c?
d. Now write up a careful proof of the statement.