Problem Set 8: Due Friday, February 23

Problem 1: Consider the unique linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ with

$$T\left(\begin{pmatrix} 9 \\ 4 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ -5 \end{pmatrix} \quad \text{and} \quad T\left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}\right) = \begin{pmatrix} -2 \\ 3 \end{pmatrix}.$$

Determine, with explanation, the value of

$$T\left(\begin{pmatrix} 6 \\ 2 \end{pmatrix}\right).$$

Problem 2: Show that the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x + 2y \\ 3x + 6y \end{pmatrix}$$

is not injective and not surjective.

Problem 3: Consider the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x \\ -x + y \end{pmatrix}.$$

Plot the values of at least 4 points and where T sends them, and then use that to describe the action of T geometrically.

Problem 4: Suppose that $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ and $S\colon \mathbb{R}^2 \to \mathbb{R}^2$ are both linear transformations. Show that $T \circ S\colon \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation.

Problem 5: Compute

$$\begin{pmatrix} 4 & 3 \\ -7 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 5 \end{pmatrix}.$$

Describe what your computation means in terms of a linear transformation. Use Problem 1 above as a guide.

Problem 6: Consider the unique linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ with

$$T\left(\begin{pmatrix} 1 \\ -1 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 4 \end{pmatrix} \quad \text{and} \quad T\left(\begin{pmatrix} -2 \\ 3 \end{pmatrix}\right) = \begin{pmatrix} 2 \\ 7 \end{pmatrix}.$$

What is $[T]$? Explain.