Problem Set 4: Due Monday, February 5

Problem 1: Consider the following statement:

$$\text{If } a \in \mathbb{Z} \text{ and } 3a+5 \text{ is even, then } a \text{ is odd.}$$

a. Write down the contrapositive of the given statement.
b. Show that the original statement is true by proving that the contrapositive is true.

Problem 2: Let $A = \{\sin x : x \in \mathbb{R}\}$.
a. In class, we talked about how we could always turn a parametric description of a set into our other description (by carving out of a bigger set) by using a "there exists" quantifier. Do that for our set A above.
b. Find another way to describe A by carving it out of a set using a property without any quantifiers. Briefly explain why your set is equal (no need to give a formal proof).

Problem 3: Determine which of the following are true or false. Briefly explain.
a. $-5 \in \mathbb{Q}$.
b. $7 \in \{1, 4, \{7\}\}$.
c. $\{1, 5\} \in \{1, 2, 6, 5\}$.
d. $\{2, 8\} \subseteq \{1, 2, 3, 4, 5, 6, 7, 8\}$.
e. $\mathbb{Z} \in \mathbb{Q}$.
f. $\{n \in \mathbb{Z} : n > \frac{1}{2}\} \cup \{n \in \mathbb{Z} : n < \frac{1}{8}\} = \mathbb{Z}$.

Problem 4: Let $A = \{1, 2, 3, 4, 5\}$, let $B = \{1, 4, 5, 7, 8, 9\}$, and let $C = \{2, 4, 6, 7, 9\}$. Determine each of the following.
a. $A \cup B$.
b. $A \cup C$.
c. $A \cap B \cap C$.
d. $(A \cup B) \cap (A \cup C)$.
e. $A \backslash (B \cup C)$.

Problem 5: Describe the set $\{x \in \mathbb{R} : x^2 - 3x - 4 > 0\}$ in another way by writing it as a union of two sets with simpler descriptions. Briefly explain why your set is equal.

Problem 6: Let $A = \{6n : n \in \mathbb{N}\} \cap \{10n : n \in \mathbb{N}\}$.
a. Write down the smallest 3 elements of A, and briefly explain how you determined them.
b. Make a conjecture about how to describe A parametrically (no need to prove this conjecture).

Problem 7: Let $A = \{12n - 7 : n \in \mathbb{Z}\}$ and let $B = \{4n + 1 : n \in \mathbb{Z}\}$.
a. Show that $B \not\subseteq A$.
b. Fill in the blanks below with appropriate phrases so that the result is a correct proof of the statement that $A \subseteq B$.

Let $a \in A$ be arbitrary. By definition of A, we can fix ______________. Now notice that $a =$ ______________.
Since ______________ $\in \mathbb{Z}$, we conclude that $a \in B$.