Problem Set 17: Due Monday, April 15

Problem 1: Working in $\mathcal{P}_3$, consider the following functions:

- $f_1(x) = x^3 + 2x^2 + x$.
- $f_2(x) = -3x^3 - 5x^2 + x + 2$.
- $f_3(x) = x^2 - x + 1$.
- $g(x) = x^3 + 8x^2 + 7$.

Is $g \in \mathsf{Span}(f_1, f_2, f_3)$? Explain.

Problem 2: Use Gaussian Elimination to classify for which values of $h, k \in \mathbb{R}$ the system

$$\begin{array}{rcrcl} x & + & hy & = & 2 \\ 4x & + & 8y & = & k \end{array}$$

has each of the following: (i) no solution, (ii) one solution, and (iii) infinitely many solutions.

Problem 3: Given $b_1, b_2, b_3 \in \mathbb{R}$, determine necessary and sufficient conditions so that

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \in \mathsf{Span}\left(\begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix} \right)$$

is true.

Problem 4: Does

$$\mathsf{Span}\left(\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) = \mathbb{R}^3?$$

Explain.

Problem 5: Let V be the vector space of all 2×2 matrices. Does

$$\mathsf{Span}\left(\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 3 \\ 7 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 2 & 6 \end{pmatrix} \right) = V?$$

Explain.

Problem 6: Let $\mathcal{D}$ be the vector space of all differentiable functions $f\colon \mathbb{R} \to \mathbb{R}$. Let $f_1\colon \mathbb{R} \to \mathbb{R}$ be the function $f_1(x) = \sin^2 x$ and let $f_2\colon \mathbb{R} \to \mathbb{R}$ be the function $f_2(x) = \cos^2 x$. Finally, let $W = \mathsf{Span}(f_1, f_2)$, and notice that W is a subspace of $\mathcal{D}$. Determine, with explanation, whether the following functions are elements of W.

a. The function $g_1\colon \mathbb{R} \to \mathbb{R}$ given by $g_1(x) = 3$.
b. The function $g_2\colon \mathbb{R} \to \mathbb{R}$ given by $g_2(x) = x^2$.
c. The function $g_3\colon \mathbb{R} \to \mathbb{R}$ given by $g_3(x) = \sin x$.
d. The function $g_4\colon \mathbb{R} \to \mathbb{R}$ given by $g_4(x) = \cos 2x$.