# Problem Set 16: Due Friday, April 12

**Problem 1:** Let $V$ be a vector space, and let $W$ be a subspace of $V$. Recall that

$$V \backslash W = \{\vec{v} \in V : \vec{v} \notin W\},$$

i.e. $V \backslash W$ is the set of elements of $V$ that are *not* in $W$. Is $V \backslash W$ always a subspace of $V$? Sometimes a subspace of $V$? Never a subspace of $V$? Explain.

**Problem 2:** Use Gaussian Elimination to solve the following system:

$$\begin{array}{rcrcrcr} x & & & - & z & = & 0 \\ 3x & + & y & & & = & 1 \\ -x & + & y & + & z & = & 4. \end{array}$$

**Problem 3:** Find the coefficients $a, b, c \in \mathbb{R}$ so that the graph of $f(x) = ax^2 + bx + c$ passes through the points $(1, 2)$, $(-1, 6)$, and $(2, 3)$.

**Problem 4:** Is

$$\begin{pmatrix} 20 \\ 0 \\ 5 \\ 10 \end{pmatrix} \in \mathsf{Span}\left( \begin{pmatrix} 0 \\ 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ -2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right)?$$

Explain.

**Problem 5:** Give a parametric description of the solution set of the following system:

$$\begin{array}{rcrcrcrcr} x & + & 2y & - & z & & & = & 3 \\ 2x & + & y & & & + & w & = & 4 \\ x & - & y & + & z & + & w & = & 1. \end{array}$$

**Problem 6:** Solve the following three systems simultaneously:

$$\begin{array}{rcrcr} x & + & 3y & = & 8 \\ 2x & + & 5y & = & 15, \end{array} \qquad \begin{array}{rcrcr} x & + & 3y & = & 3 \\ 2x & + & 5y & = & 4, \end{array} \qquad \text{and} \qquad \begin{array}{rcrcr} x & + & 3y & = & -10 \\ 2x & + & 5y & = & -14. \end{array}$$

Notice that the coefficients of $x$ and $y$ are the same, so you should code these simultaneously as one matrix, and go to reduced echelon form (see p. 181-182 of the book).