Problem Set 13: Due Monday, March 11

Problem 1: Find the eigenvalues of the matrix

$$\begin{pmatrix} 5 & -1 \\ -7 & 3 \end{pmatrix}.$$

Problem 2: Find the eigenvalues of the matrix

$$\begin{pmatrix} 1 & 8 \\ 2 & 1 \end{pmatrix},$$

and then find (at least) one eigenvector for each eigenvalue.

Problem 3: Find the eigenvalues of the matrix

$$\begin{pmatrix} 2 & -1 \\ 1 & 4 \end{pmatrix},$$

and then find (at least) one eigenvector for each eigenvalue.

Problem 4 Find values for c and d such that the matrix

$$\begin{pmatrix} 3 & 1 \\ c & d \end{pmatrix}$$

has both 4 and 7 as eigenvalues. You should show the derivation for how you arrived at your choice.

Problem 5: Consider the unique linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ with

$$[T] = \begin{pmatrix} 1 & 0 \\ 6 & -1 \end{pmatrix}.$$

Determine whether T is diagonalizable. If so, find an example of a basis $\alpha = (\vec{u}_1, \vec{u}_2)$ of $\mathbb{R}^2$ such that $[T]_\alpha$ is a diagonal matrix, and determine $[T]_\alpha$ in this case.

Problem 6: Consider the unique linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ with

$$[T] = \begin{pmatrix} 3 & -1 \\ 1 & 5 \end{pmatrix}.$$

Determine whether T is diagonalizable. If so, find an example of a basis $\alpha = (\vec{u}_1, \vec{u}_2)$ of $\mathbb{R}^2$ such that $[T]_\alpha$ is a diagonal matrix, and determine $[T]_\alpha$ in this case.