Problem Set 1: Due Friday, January 26

Problem 1: For each part, explain your reasoning using a sentence or two.
a. Consider the line in the plane described by the equation $3x - 2y = 12$. Find an example of $a, b, c, d \in \mathbb{R}$ such that

$$\begin{aligned} x &= a + bt \\ y &= c + dt \end{aligned}$$

is a parametric equation for the line.
b. Find two other choices for $a, b, c, d \in \mathbb{R}$ that work for part (a).
c. Consider the line in the plane described parametrically by

$$\begin{aligned} x &= 2 - 3t \\ y &= 1 + 5t. \end{aligned}$$

Using this parametric description, find a point on the line and the slope of the line.
d. Find an example of $a, b, c \in \mathbb{R}$ such that the line in part (c) is described by the equation $ax + by = c$.
e. Find another example of $a, b, c \in \mathbb{R}$ that works for the line in part (c).

Problem 2: Let P be the plane in $\mathbb{R}^3$ that contains the origin and that is parallel to each of the following two vectors:

$$\vec{u} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \qquad \text{and} \qquad \vec{w} = \begin{pmatrix} -7 \\ 1 \\ 4 \end{pmatrix}$$

In the book, we discussed one way to parametrize P, and we will discuss this in more detail later. Now find an equation of the form $ax + by + cz = d$ for P. Explain your process using a sentence of two.

Problem 3: Let L be the line in $\mathbb{R}^3$ that is the intersection of the two planes $3x + 4y - z = 2$ and $x - 2y + z = 4$.
a. Using the equations of the planes, determine if the points $(1, 0, 1)$ and $(1, 1, 5)$ are on L.
b. Find a parametric description of L. Explain your process using a sentence or two.
c. Use the parametric description of L to determine if $(5, 2, 3)$ is a point on L. Explain.
Note: Given a point, it seems easier to determine if it is on L using the equations of the planes rather than the parametric description. In contrast, if you want to *generate* points on L, it is easier to use the parametric description (just plug in values for the parameter) than the plane equations.

Problem 4: Define a function $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ by letting

$$f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x - y \\ x + y \end{pmatrix}.$$

Think of f as transforming the plane as we discussed in class, and as illustrated on p. 9 of the book. As discussed there, it appears that f rotates the plane 45^o counterclockwise while simultaneously scaling the plane by a factor of $\sqrt{2}$. In this problem, you will verify some of these statements.
a. Show that for all $\vec{v} \in \mathbb{R}^2$, we have $||f(\vec{v})|| = \sqrt{2} \cdot ||\vec{v}||$, where $||\vec{v}||$ is the length of $\vec{v}$.
b. Use the dot product to show that for all nonzero $\vec{v} \in \mathbb{R}^2$, the angle between $\vec{v}$ and $f(\vec{v})$ is 45^o.

Problem 5: Are the following statements true or false? Explain your reasoning in each case.
a. There exists $x \in \mathbb{Z}$ with $x^3 - 8x = 3$.
b. There exists $x \in \mathbb{R}$ with $\sin x + \cos x = 3$.