## Writing Assignment 1: Due Wednesday, January 29

**Problem 1:** Determine whether the following statements are true or false. In all cases, explain your reasoning thoroughly in complete sentences.
a. There exists $m, n \in \mathbb{Z}$ such that $34m + 30n = 2$.
b. There exists $m, n \in \mathbb{N}$ such that $5m + 9n = 16$.
c. For all $a \in \mathbb{R}$, we have $a^6 - 4a^3 + 9 \geq 3$.

**Problem 2:** Define a function $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ by letting

$$f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x - y \\ x + y \end{pmatrix}.$$

Think of $f$ as transforming the plane as we discussed in class, and as illustrated on p. 9 of the course notes. As discussed there, it appears that $f$ rotates the plane $45^o$ counterclockwise and simultaneously scales the plane by a factor of $\sqrt{2}$. In this problem, you will verify some of these statements.
a. Show that for all $\vec{v} \in \mathbb{R}^2$, we have $||f(\vec{v})|| = \sqrt{2} \cdot ||\vec{v}||$, where $||\vec{v}||$ is the length of $\vec{v}$.
b. Show that for all nonzero $\vec{v} \in \mathbb{R}^2$, the angle between $\vec{v}$ and $f(\vec{v})$ is $45^o$.
c. Parts (a) and (b) give two possibilities of $f(\vec{v})$, since it might be clockwise or counterclockwise relative to $\vec{v}$. Think about how you check that it is counterclockwise, and try to verify it.
*Hint:* Think about using some of the vector operations you learned in Calculus II for parts (b) and (c).